Supplementary Active Stabilization of Nonrigid Gravity Gradient Satellites

JAMES E. KEAT*
Westinghouse Systems Development Division, Baltimore, Md.

Theme

REACTION jets are one of the methods for augmenting the dynamics of satellites whose attitude nominally is stabilized by gravity gradient (GG) forces and a passive damper. The present study differs from previous ones on attitude control by jets in two main ways: 1) It includes nonrigidity, and 2) it uses the Hamiltonian H to develop the control laws. Consideration of nonrigidity is necessary when active control forces are applied to vehicles with flexible parts, such as the inertia booms usually needed on GG satellites. Previous use of H in satellite studies has been concerned largely with its application as a Lyapunov function in stability investigations of passive vehicles.

Contents

Let H be the Hamiltonian of the GG satellite's state $\mathbf{x}$ under the Keplerian orbit approximation. $\mathbf{x}$ defines the attitude displacement and rate relative to the local vertical-orbit pole cartesian frame R and, if the satellite is nonrigid, the relative motions of its parts. The potential energy terms included in H result from internal stiffness and the first-order component of the central force gravity field. Assume that the satellite contains no parts whose motions are not constrained by stiffness forces, that no active alteration of the mass distribution is performed, and that all three principal moments of inertia are unequal. If disturbances Q_{ds} and orbit eccentricity e are negligible, it then will possess time-invariant stable equilibrium states $\mathbf{x}_{se}$. A constant K_u can be added to make $H = 0$ when $\mathbf{x} = \mathbf{x}_{se}$. Then $H > 0$ when $\mathbf{x} \neq \mathbf{x}_{se}$. H thus can serve as a measure of the displacement of $\mathbf{x}$ from $\mathbf{x}_{se}$. If internal damping forces Q_{dm} are negligible, H is constant when Q_{ds}, e, and the jet forces $\mathbf{u}$ are zero, since it then is not an explicit function of time. Q_{dm} yields $\dot{H} \leq 0$. Q_{ds} and e can produce bounded variations in H and, under some conditions, secular growth. The purpose of $\mathbf{u}$ can be regarded as being to supplement Q_{dm} by reducing H or maintaining it small. Depending on the application, it may be necessary to supplement control criteria based on H by constraints to prevent alteration of the orbit or to limit the magnitudes of critical structural vibration modes. If the satellite is tumbling at the start of the jet operation and not all the $\mathbf{x}_{se}$'s are suitable for the mission, a constraint is needed on the $\mathbf{x}_{se}$ about which capture is achieved.

The total time derivative $\dot{H}$ of H can be put in the following form

$$\dot{H} = \dot{H}_u + \dot{H}_{dm} + \dot{H}_{ds} \tag{1}$$

where

$$\dot{H}_u = \mathbf{v}_u'\mathbf{u} = \sum_{\alpha=1}^{s} v_{u\alpha}u_\alpha \tag{2}$$

The elements u_α of the $s \times 1$ control vector $\mathbf{u}$ are the forces of the individual jets. The elements $v_{u\alpha}$ of the $s \times 1$ velocity vector $\mathbf{v}_u$ are the velocities of the jets, relative to R, along their force axes. In the present paper, " ′ " indicates the transpose of a column vector or matrix. $\dot{H}_u$ and $\dot{H}_{dm}$ are the components of $\dot{H}$ due to $\mathbf{u}$ and Q_{dm}, respectively. The remaining components have been combined into $\dot{H}_{ds}$. The time integral of $\dot{H}_u$ is the work done, relative to R, by $\mathbf{u}$. The study will employ the tenet that no jet ever should act directly to increase H. When any jet α is fired, Eq. (2) then shows that the proper polarity of its u_α is opposite to $v_{u\alpha}$ and that its effectiveness in reducing H is proportional to the magnitude $|v_{u\alpha}|$ of $v_{u\alpha}$. A simple control policy then consists of 1) energizing each jet when its $|v_{u\alpha}|$ reaches a selected value or else is close to a local maximum, and 2) de-energizing it when $|v_{u\alpha}|$ reaches zero. This policy can be supplemented as necessary by the aforementioned control constraints. If the condition is reached in which all $v_{u\alpha}$ remain zero, no further reduction of H by the jets is possible, since any remaining motions are in uncontrollable modes.

A model of nonrigid GG satellite dynamics is needed for study of the time response of $\mathbf{v}_u$. A convenient general model can be obtained by modifying equations given in Ref. 1 into the form

$$\mathbf{M}_v\dot{\mathbf{v}} = \mathbf{Y}_v\mathbf{u} - \mathbf{N}_v\mathbf{v} + \mathbf{f} \tag{3}$$

Detailed equations for the terms in Eq. (3) are presented in the backup paper. The relative displacements of the satellite's parts and its attitude relative to R are modeled by an $m \times 1$ generalized coordinate vector $\mathbf{q}$. $\mathbf{v}$ is an $m \times 1$ generalized velocity vector. $\mathbf{M}_v$, $\mathbf{Y}_v$, $\mathbf{N}_v$, and $\mathbf{f}$ are, or can be, functions of $\mathbf{q}$. $\mathbf{M}_v$ is an $m \times m$ generalized mass matrix. $\mathbf{Y}_v$ is an $m \times s$ matrix which converts $\mathbf{u}$ into an $m \times 1$ generalized control vector. $-\mathbf{N}_v\mathbf{v}$ consists of the internal viscous damping forces and the gyroscopic forces due to the mean anomaly rate. $\mathbf{f}$ is comprised of the remaining forces and "apparent forces," including internal stiffness. $\mathbf{v}_u$ and $\mathbf{v}$ can be shown to be related through

$$\mathbf{v}_u = \mathbf{Y}_v'\mathbf{v} \tag{4}$$

As an example of the approach, the case will be considered in which the jet outputs can be approximated as impulses. This is the simplest case to study because $\mathbf{q}$, and hence $\mathbf{M}_v$ and $\mathbf{Y}_v$, are constant during any impulse and because the step change in $\mathbf{v}$ is not influenced by $\mathbf{N}_v\mathbf{v}$ or $\mathbf{f}$. Let the satellite contain s jets. Let $\mathbf{u}_I$ be the $s \times 1$ vector of the impulses when one or more jets are fired at a time t_0. It will be convenient to introduce into the mathematics a reduced-order controller, designated by "a," through the equation $\mathbf{u}_I = \mathbf{C}\mathbf{u}_I^a$. $\mathbf{u}_I^a$ has dimensions $s_a \leq s$. This constraint is intended to encompass the cases where 1) the jets are employed in one or more sets so that the elements of $\mathbf{u}_I$ are not all independent, and/or 2) all s jets are not used in a single firing. $\mathbf{C}$ is constant during any impulse but will differ in each firing depending on the jets that are used. $s_a = 1$ if the control policy outlined earlier is used, since the jets or jet sets are fired individually. Define $\mathbf{v}_u^a = \mathbf{C}'\mathbf{v}_u$. Using Eqs. (1–3), the step change ΔH in H can be shown to be

$$\Delta H = \mathbf{v}_{u0}^{a\prime}\mathbf{u}_I^a + .5\mathbf{u}_I^{a\prime}\mathbf{D}_a\mathbf{u}_I^a \tag{5}$$

where $\mathbf{D}_a = \mathbf{C}'\mathbf{D}\mathbf{C}$ and $\mathbf{D} = \mathbf{Y}_v'\mathbf{M}_v^{-1}\mathbf{Y}_v \cdot \mathbf{v}_{u0}^a$ is $\mathbf{v}_u^a$ at $t_0(-)$. Each element $D_{\alpha\beta}$ of $\mathbf{D}$ is the step change $\Delta v_{u\alpha}$ at jet α generated by a unit impulse from jet β.

Received June 12, 1972; synoptic received September 12, 1972. Full paper is available from National Technical Information Services, Springfield, Va., 22151, as N73-12904 at the standard price (available upon request). This work was supported in part by Goddard Space Flight Center under Contracts NAS5-11733-5(RAE) and NAS5-11734-26(RAE).

Index category: Spacecraft Attitude Dynamics and Control.

* Now Senior Member of the Technical Staff, Computer Sciences Corporation, Silver Spring, Md. Member AIAA.

The optimum impulse vector $\mathbf{u}_I^{a*}$ is the one which minimizes ΔH. Using Eq. (5), it can be shown that

$$\mathbf{u}_I^{a*} = -\mathbf{D}_a^{-1}\mathbf{v}_{u0}^{a} \tag{6}$$

$\mathbf{u}_I^{a*}$ drives $\mathbf{v}_u^a$ to zero. Nonsingularity of $\mathbf{D}_a$ can be assured by proper choice of $\mathbf{C}$.

Equation (6) provides the basis for a simple control technique. Although it includes structural nonrigidity, nonrigidity can produce two difficulties. First, even if the mathematical model of the system can be considered perfect, the first one or more $\mathbf{u}_I^{a*}$ impulses may generate unacceptably large structural vibrations. This can occur, inspite of the fact that H is reduced, if the initial angular rate is large. The second difficulty concerns the validity of the mathematical model, particularly the approximation of the jet outputs by impulses and representation of nonrigidity via a finite-dimensional coordinate vector $\mathbf{q}$. The $\mathbf{u}_I^{a*}$'s computed by Eq. (6) are dependent on the nonrigidity model, since this determines $\mathbf{D}$. Supplementary study may be needed to verify that the accuracy of the computed $\mathbf{D}$ is not degraded intolerably by nonrigidity effects or modes not included in $\mathbf{q}$ and that the excitation of these modes is acceptably small. Studies made on the flexible satellite RAE-B showed that $\mathbf{u}_I^{a*}$'s computed by Eq. (6) would not excite unacceptable vibrations because the satellite/jet configuration limited the $\mathbf{u}_I^{a*}$ elements to small values. The preliminary results indicate that Eq. (6), with $\mathbf{D}$ established by a model that includes only fundamental vibration modes, should be adequate for sizing the jet pulses. The RAE-B study is described in Ref. 2.

Eq. (6) cannot predict the $v_{u\alpha} = 0$ jet cutoff times in problems where the impulse approximation is not adequate. The following dynamical model, obtained by linearizing Eqs. (3) and (4) and transforming to state variables, is suitable for such studies

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{g}_1 + \mathbf{G}_u\mathbf{u} \tag{7}$$

$$\mathbf{v}_u = \mathbf{B}\mathbf{x} \tag{8}$$

$\mathbf{x}$ is the $2m \times 1$ state vector. $\mathbf{A}$, $\mathbf{B}$, $\mathbf{G}_u$, and $\mathbf{g}_1$ are constant over the region of validity of the equations. The backup paper discusses the linearization procedure, presents the details of the equations, and investigates their use to determine the $v_{u\alpha} = 0$ points when the jet outputs are rectangular pulses.

For some systems, it may, at times, be necessary to override the $v_{u\alpha} = 0$ jet cutoff policy in order to prevent the amplitudes of critical vibration modes from exceeding specified values. For prediction of these corrected cutoff times, it is convenient to convert Eq. (7) into normal modes. This yields equations of the form

$$\dot{z}_\xi = \lambda_\xi z_\xi + g_{1\xi} + \mathbf{g}_{u\xi}'\mathbf{u} \tag{9}$$

where $\xi = 1$ to $2p$. p is the number of critical modes. The z_ξ's are the modal coordinates. The λ_ξ's are the eigenvalues. Assume that the jet outputs are constant-amplitude pulses and that internal damping can be neglected. Eqs. (9) then can be integrated once, over intervals of constant $\mathbf{u}$, to yield equations for the real (r) components of the motions. The results can be placed in the form

$$\tilde{v}_{vr}^{\,2} + (\tilde{z}_{vr} - \mathbf{b}_v'\mathbf{u})^2 = K_v^{\,2} \tag{10}$$

where $v = 1$ to $2p$. The K_v's are the constants of integration. The $\tilde{z}_{vr}$'s and $\tilde{v}_{vr}$'s are modified modal coordinates and velocities whose exact definitions are given in the backup paper. The effect of $\mathbf{u}$ is to shift temporarily the equilibrium points. Sensitivity of certain modes, such as those involving mainly c.m. translation rather than rotation about the c.m., may be curtailed by employing the jets properly in sets so that $\mathbf{b}_v'\mathbf{u}$ is reduced. The mode amplitude constraints can be expressed in the form

$$\tilde{v}_{vr}^{\,2} + \tilde{z}_{vr}^{\,2} \le L_v^{\,2} \tag{11}$$

where the L_v's are the selected limits. Equations (10) and (11) can be used to predict corrected shutoff times. This is discussed in the backup paper. $\tilde{z}_{vr} - \tilde{v}_{vr}$ phase plane plots are useful for visualizing the motions.

References

[1] Keat, J. E., "Dynamical Equations of Nonrigid Satellites," *AIAA Journal*, Vol. 8, No. 7, July 1970, pp. 1344–1345.

[2] Pleasants, W., "Post Deployment Use Of Thrusters On RAE-B," Final Report NAS5-11734-26, Task 3, Aug. 1972, Westinghouse Systems Development Div., Baltimore, Md.